LONGITUDINAL ANALYSIS OF FRIENDSHIP NETWORKS

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Abstract

This note discusses and demonstrates methods, both exploratory and confirmatory, for analyzing data from friendship networks collected over time. The focus is on stochastic models for dyadic interaction designed to quantify the structural effect of reciprocity on arc changes. The networks studied were previously analyzed by Hallinan (Social Networks, 1:193-210) who was concerned with stability of dyadic choices and the direction of change of asymmetric dyads to either mutual or null dyads. These aspects of the networks are really of secondary importance to the effect of reciprocated choices on the probabilities of dyadic change. Measures of this "reciprocity effect" are presented, and comments on the rationale for continuous-time Markov chains as models for networks are given.

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Introduction

In a recent issue of Social Networks, Hallinan (1978) studies friendship formation in elementary school classrooms as a continuous-time stochastic process. She presents a "friendship model," a Markov chain for the dyadic interaction between two individuals, and investigates the stability of choices over time and direction of change of asymmetric dyads to either mutual or null dyads. As expected, asymmetric dyads consisting of a single unreciprocated choice have a much shorter duration than mutual, reciprocated dyads, and surprisingly asymmetric dyads are more likely to become null than mutual dyads over time. Little research on the dynamics of networks has been conducted[1] so that her contribution is certainly welcome, and her findings, quite interesting. However, an important aspect of the process of friendship formation is overlooked in the analysis.

First, Hallinan's analysis is an exploratory longitudinal analysis. She does not adopt a statistical model incorporating structural parameters that can be properly estimated and evaluated in a confirmatory manner. We and our colleagues have developed several models, have discussed their statistical properties, and have demonstrated their usefulness. Of particular relevance to dyadic interaction is our model for reciprocity (Holland and Leinhardt 1977, and Wasserman 1979b) that includes four parameters measuring the effect that reciprocated arcs have on the probabilities of dyadic change. Our second point is that these parameters are of primary sociological significance. The probabilities and rates for stability and direction of change studied by Hallinan are of secondary importance. What could be more relevant to the investigation of dyadic interaction than the effect that a choice $j \rightarrow i$ has on whether or not $i \rightarrow j$? Am I more likely to choose you as a friend if I am your friend? And am I less likely to withdraw my friendship from you if I am your friend? Hallinan unfortunately did not answer these questions.

Reciprocity is one of the most elementary structural effects, operating at the low "level" of the dyad. Our reciprocity model provides a decomposition of the empire network of g individuals into a collection of $\binom{g}{2}$ independent and identically distributed "dyad processes." Knowledge of the mathematics of the model is not necessary for this very general discussion, but those interested should refer to Wasserman (1979b).

Longitudinal analysis is not always straightforward as stochastic models for networks are still in their infancy. In this note, we point out some of their advantages and disadvantages, and stress some of the pitfalls of this methodology so that they can be avoided by future modellers.[2] We discuss Markov models and longitudinal data in the next sections, and then give some suggestions for exploratory longitudinal analyses. Lastly, we illustrate our methods on several of Hallinan's classrooms.

Longitudinal Models

There is a common belief that continuous-time stochastic processes are superior to discrete-time processes for modelling social phenomena. While it is true that most social processes evolve continuously, the mathematics of continuous-time stochastic models make their usage quite limited. The main problem arises from the lack of a continuous record of the phenomenon under study. At best, a researcher may observe a process several times. Rarely are continuous event histories collected. We have yet to find mention of a continuous record of a network in the literature. Recent research on the estimation problems caused by this situation, most notably Keiding (1974, 1975), Singer and Spilerman (1974, 1976, 1977), and Singer and Cohen (1978), is quite promising; but for the present, continuous-time modelling of processes with just a few data observations is a difficult problem.

We advocate greater use of discrete-time models for longitudinal network studies. Do friendships actually change minute-to-minute, or are daily, or even weekly changes in attitude more realistic? Discrete-time Markov chains are easy to understand and analyze. Statistical tests of stationarity, order, and population homogeneity are well developed and powerful against broad classes of alternatives. In addition, waiting times are geometric, rather than exponential, random variables, so that stability can be easily studied. We have found discrete-time Markov chains useful and insightful as models for a corporate network (Galaskiewicz and Wasserman, 1979).

The implication that a network, or some function of a network, evolves as a continuous-time Markov chain is strong. The process remains in a specified state for an exponentially distributed length of time, with a rate dependent only on the state. Immediately thereafter, a transition is made to another state, with a probability that depends only on the present and transited state.[3] Do networks change in this fashion? A Monte Carlo simulation of this process, comparing the modelled evolution and resulting equilibrium to what we currently know about the empirical dynamics of friendship networks would be quite valuable.

Longitudinal Data

If a network of g individuals is observed or questioned regarding friendship preferences at n different times, $t_1, t_2, \ldots, t_n$, then we can form n sociomatrices, $\underset{\sim}{X}(t_1), \underset{\sim}{X}(t_2), \ldots, \underset{\sim}{X}(t_n)$. In later sections of this note, we will be examining three of the five classrooms studied by Hallinan (1978), Classes 1, 2, and 3 by her numbering. All are sixth grades, and each is observed at $n = 7$ equally-spaced times. The time between sociometric questioning is a constant 42 days. We will let $\underset{\sim}{X}$, $\underset{\sim}{Y}$, and $\underset{\sim}{Z}$ denote the set of sociomatrices for Classes 1, 2, and 3, respectively.

To study dyadic change, these 21 sociomatrices are reduced to records of the off-diagonal pairs $[X_{ij}(t), X_{ji}(t)]$, $[Y_{ij}(t), Y_{ji}(t)]$, and $[Z_{ij}(t), Z_{ji}(t)]$, for all seven time points, where as usual

$$X_{ij}(t) = \begin{cases} 0 \text{ if } i \rightarrow j \text{ at time } t \\ 1 \text{ if } i \rightarrow j \text{ at time } t \end{cases}$$

and similarly for the elements of Y and Z. If we pick a pair of time points (t_k, t_ℓ), $t_k < t_\ell$, then for each classroom, we can form a 3×3 cross-classified table of frequencies of dyadic transitions. There are $\binom{n}{2}\times 3 =$ 63 such tables for our three classrooms. Each table contains 10 distinct counts, as illustrated in Table 1.

Note that these tables are <u>not</u> square, but contain 10 rather than $3\times 3 = 9$ entries. That extra piece of information is $A \Rightarrow A^*$, the number of asymmetrics that reverse direction. It is not possible to analyze such a nonsquare Markov transition matrix.[4] We and Hallinan ignore these asymmetric reversals, and simply consider the dyadic arc process, with states defined by the number of arcs in the dyad, 0, 1, or 2. It is important to stress that if there are a substantial number of such reversals, and there

<table>
<tr><td></td><td></td><td colspan="3">Time t_ℓ</td></tr>
<tr><td></td><td></td><td>Null</td><td>Asymmetric</td><td>Mutual</td></tr>
<tr><td rowspan="4">Time t_k</td><td>Null</td><td>$N \Rightarrow N$</td><td>$N \Rightarrow A$</td><td>$N \Rightarrow M$</td></tr>
<tr><td rowspan="2">Asymmetric</td><td rowspan="2">$A \Rightarrow N$</td><td>$A \Rightarrow A$ same direction</td><td rowspan="2">$A \Rightarrow M$</td></tr>
<tr><td>$A \Rightarrow A^*$ reverse direction</td></tr>
<tr><td>Mutual</td><td>$M \Rightarrow N$</td><td>$M \Rightarrow A$</td><td>$M \Rightarrow M$</td></tr>
</table>

Table 1. Table of Dyadic Transitions ($\Rightarrow$ indicates transition) e.g., $N \Rightarrow N$ = number of dyads that were null at times t_k and t_ℓ.

is likely to be if t_k and t_ℓ are far apart in time, then reduction of the dyad process to the arc process implies a substantial loss in information. Fortunately, there are very few reversals in the three classes.

Friendship networks are notoriously sparse, with very low densities. Between 60 and 70 percent of the choices in the three networks are zeros. Another important question is how sensitive are our longitudinal methods to network density? With so many null relationships, some of our findings could be artifactual. We will discuss this further in a later section.

Exploratory Analysis

The initial analysis of these classrooms was essentially exploratory.[5] No explicit model for the $\underset{\sim}{Q}$ intensity matrix was assumed, so that only a qualitative description of its elements was given. A thorough exploratory analysis involves not only a verification of Markov assumptions but also a study of the standard network statistics on the cross-sectional data. Holland (1977) outlines a good cross-sectional exploratory analysis of sociometric data.

The reciprocity model incorporates three assumptions:

(1) Movement to new states is Markovian;

(2) Transition probabilities are stationary;

(3) Dyads are homogeneous; i.e., have identical transition rates.

Assumptions (1) and (2) are easy to check; Anderson and Goodman (1957) and Billingsley (1961) discuss hypothesis tests of these assumptions. Population homogeneity, Assumption (3), is difficult to test. All we can hope is that the individuals are similar enough across a variety of traits that the set of dyads are identically distributed. We have done some work on testing for homogeneity in Poisson processes (Wasserman, 1978b) and Singer and Spilerman (1978) discuss heterogeneous mixtures of mobility models, but more research is needed for general stochastic processes.

One source of population heterogeneity in friendship networks is differential popularity. If individuals i and j are low in popularity, and i' and j' are high, are the probabilities of dyadic change identical for the dyads involving i and j, and i' and j'? And what of the change rates for the high-low dyads, i' and j and j' and i? Hallinan (1978), Newcomb (1961), and many other sociologists have also posed these questions. We will come back to them in the last section.

In testing for Markovian transitions, we must check the following equality for all triples of time points, $t_k < t_\ell < t_m$:

$$\hat{\underset{\sim}{P}}(t_k,t_m) = \hat{\underset{\sim}{P}}(t_k,t_\ell)\hat{\underset{\sim}{P}}(t_\ell,t_m)$$

where $\hat{\underset{\sim}{P}}$ is the empirical probability transition matrix for a pair of time points.[6] With n time points, there are $\binom{n}{3}$ $\hat{\underset{\sim}{P}}$ matrices to study. With so many hypothesis tests, it is important to fix an experimentwise error rate to guard against rejecting too many of the null hypotheses. The standard procedure is to use Bonferonni's inequality and compare p-values not to a predetermined α, but to $\frac{\alpha}{T}$, where T = the number of simultaneous hypothesis tests (see Miller, 1977).

It is nonsensical to test continuous-time Markov chains for order. The notion of order is relevant only to discrete-time chains, where we can talk of conditioning observations on previous generations. What is relevant, is time-stationarity, Assumption (2). For all pairs of time points, $s < t$, the equality

$$\underset{\sim}{P}(s,t) = \underset{\sim}{P}(t-s)$$

must hold, i.e., probability transition matrices must depend only on the length of the transition interval, and not when the interval began.

If we do not reject the hypothesis of stationarity, then we can combine all the transition matrices for a given class to form a "super" transition matrix, since our observations are equally spaced. If the elements of Table 1 for two time points $t_k < t_\ell$, are denoted by $n_{ij}(t_k, t_\ell)$, then this super empirical transition matrix has elements

$$\hat{p}_{ij} = \sum_{k=1}^{n-1} n_{ij}(t_k, t_{k+1}) / \sum_{j=1}^{3} \sum_{k=1}^{n-1} n_{ij}(t_k, t_{k+1}).$$

Note that the above equation is a pooling of the n_{ij}, not an averaging of the $\hat{p}_{ij}(t_k, t_\ell)$. The pooled counts form the maximum likelihood estimates, which are not equivalent to the average of the $\hat{p}_{ij}$, unless the rowsums of the n_{ij} are all equal.

Estimating Reciprocity Efforts

Once we have verified that the data adhere to the three implicit assumptions of a continuous-time stationary Markov chain, we can ask whether or not the dyads behave according to the reciprocity model. The model postulates a $\underset{\sim}{Q}$ intensity matrix, shown in Table 2, based on the following probabilities:

$$P\{X_{ij}(t+h) = 1 | X_{ij}(t) = 0, X_{ji}(t) = 0\} = \lambda_0$$
$$P\{X_{ij}(t+h) = 0 | X_{ij}(t) = 1, X_{ji}(t) = 0\} = \lambda_1$$
$$P\{X_{ij}(t+h) = 1 | X_{ij}(t) = 0, X_{ji}(t) = 1\} = \lambda_0 + \mu_0$$
$$P\{X_{ij}(t+h) = 0 | X_{ij}(t) = 1, X_{ji}(t) = 1\} = \lambda_1 + \mu_1$$

A confirmatory analysis of the model entails an estimation of $\underset{\sim}{Q}$, and thus the parameters, via maximum likelihood subject to the restriction that $q_{NM} = q_{MN} = 0$, and testing whether $\underset{\sim}{\hat{P}}$ is sufficiently close to $\underset{\sim}{\tilde{P}} = e^{t\underset{\sim}{\tilde{Q}}}$, via χ^2 goodness-of-fit tests, where $\underset{\sim}{\tilde{Q}}$ is the MLE of $\underset{\sim}{Q}$.[7] Greater details are given in Wasserman (1979a).

		Time t+h		
		Null	Asymmetric	Mutual
	Null	$-2\lambda_0$	$2\lambda_0$	0
Time t	Asymmetric	λ_1	$-(\lambda_0+\lambda_1+\mu_0)$	$\lambda_0+\mu_0$
	Mutual	0	$2(\lambda_1+\mu_1)$	$-2(\lambda_1+\mu_1)$

Table 2. $\underset{\sim}{Q}$, the intensity matrix for the reciprocity model.

Both we and Hallinan study the embeddability of the super empirical transition matrix $\hat{\underset{\sim}{P}}$ as arising from a continuous-time Markov chain, by examining $\hat{\underset{\sim}{Q}} = \log \hat{\underset{\sim}{P}}$. This is the endpoint of Hallinan's analysis, but just the beginning of ours. $\hat{\underset{\sim}{Q}}$ does not yield maximum likelihood estimates of the parameters. One must use the elements of $\hat{\underset{\sim}{Q}}$ as starting values for a grid search of the four dimensional likelihood surface.

It is well known that the logarithmic function of $\hat{\underset{\sim}{P}}$ is one-to-one, and consequently yields a unique $\hat{\underset{\sim}{Q}}$, when the eigenvalues of $\hat{\underset{\sim}{P}}$ are real, distinct, and nonnegative. In the context of the reciprocity model, $\hat{\underset{\sim}{P}}$ has three eigenvalues, one of which is unity. The remaining two eigenvalues are very likely to be distinct, real, and positive if the diagonal elements of $\hat{\underset{\sim}{P}}$ are large relative to the off-diagonal elements. This "diagonal-dominant" situation is obtained when the time interval between observations is small, so that few changes of state have occurred. What is still unknown at present is the effect that small perturbations of $\hat{\underset{\sim}{P}}$ have on the calculation of $\hat{\underset{\sim}{Q}}$. Taking matrix logarithms is a sensitive procedure. Hallinan ascribes the lack-of-fit found in her exploratory analysis to data "noise"; if this truly is the case, then her findings must be given only trivial consideration because of this sensitivity.

Fortunately, our estimation procedure relies very little on the calculation of such logarithms. The structural parameter estimates are given in Table 3.

	Class 1 $g = 26$	Class 2 $g = 28$	Class 3 $g = 30$
$\hat{\lambda}_0$	0.097	0.076	0.048
$\hat{\lambda}_1$	0.530	0.450	0.480
$\hat{\mu}_0$	0.190	0.210	0.370
$\hat{\mu}_1$	-0.290	-0.250	-0.280
$\hat{\lambda}_0 + \hat{\mu}_0$	0.287	0.286	0.428
$\hat{\lambda}_1 + \hat{\mu}_1$	0.240	0.200	0.200
$\hat{\rho}_1 = \dfrac{\hat{\lambda}_0 + \hat{\mu}_0}{\hat{\lambda}_0}$	2.959	3.763	8.917
$\hat{\rho}_2 = \dfrac{\hat{\lambda}_1 + \hat{\mu}_1}{\hat{\lambda}_1}$	0.453	0.444	0.417

Table 3. Maximum Likelihood Estimates of Reciprocity Model Structural Parameters

The parameters λ_0 and λ_1 are the probabilities, in the absence of reciprocated choices, of asymmetric choices being formed (λ_0) and asymmetric choices being withdrawn (λ_1) in the next moment of time. Values near unity indicate good possibilities for these arc changes while values near zero imply little chance of these changes. These parameters are remarkably constant across the classrooms. There is little chance of choices being made in the absence of reciprocated choices ($\hat{\lambda}_0$ = 0.097, 0.076, 0.048); however, choices have a good chance of being withdrawn if reciprocated choices are not present ($\hat{\lambda}_1$ = 0.530, 0.450, 0.480).

The parameter sums $\lambda_0 + \mu_0$ and $\lambda_1 + \mu_1$ are the probabilities of a choice $i \rightarrow j$ forming or disappearing in the next moment of time given that the reciprocated choice $j \rightarrow i$ is present. These probabilities are intermediate in magnitude between $\hat{\lambda}_0$ and $\hat{\lambda}_1$, falling between 0.2 and 0.5. Classes 1 and 2 are very similar; the probabilities of arcs forming, given reciprocity, are roughly $\hat{\lambda}_0 + \hat{\mu}_0 = 0.29$, while those for arcs disappearing are roughly $\hat{\lambda}_1 + \hat{\mu}_1 = 0.20$. Class 3 has a greater chance for new choices given reciprocated choices ($\hat{\lambda}_0 + \hat{\mu}_0 = 0.428$) than the other two classes, and about the same chance for withdrawn choices ($\hat{\lambda}_1 + \hat{\mu}_1 = 0.20$) as the other classes. In brief, there is virtually no tendency for choices to be made, and a moderate chance that choices will be withdrawn, without reciprocity, in all classes. In addition, the chance that choices will be made in class 3 with reciprocity is substantial.

It is best to interpret these parameters in relation to each other. A comparison of λ_0 to $\lambda_0 + \mu_0$ gives the effect of a reciprocated choice on the formation of new choices. A comparison of λ_1 to $\lambda_1 + \mu_1$ tells what effect a reciprocated choice has on the withdrawal of old choices. Hence, the <u>reciprocity ratios</u> $\rho_1 = (\lambda_0 + \mu_0)/\lambda_0$ and $\rho_2 = (\lambda_1 + \mu_1)/\lambda_1$, summarize quite well the information contained in the dyad processes. The estimates of these ratios are shown at the bottom of Table 3.

The ratio $\hat{\rho}_1$ takes on the values 2.959, 3.763 and a whopping 8.917 in classes 1,2, and 3. Reciprocated arcs have a large influence on the appearance of new choices; for example, a choice is nearly 9 times more likely to be made in the presence of reciprocity than in its absence in class 3. The ratio $\hat{\rho}_2$ is roughly 0.43 across the three classes, and indicates that choices are less than half as likely to be withdrawn in the presence of reciprocity.

The reciprocity parameter estimates are functions of the elements of the maximum likelihood estimate of $\underset{\sim}{Q}$. Specifically,

$$\hat{\lambda}_0 = \frac{1}{2}\tilde{q}_{NA} = -\frac{1}{2}\tilde{q}_{NN}$$

$$\hat{\lambda}_1 = \tilde{q}_{AN}$$

$$\hat{\lambda}_0+\hat{\mu}_0 = \tilde{q}_{AM}$$

$$\hat{\lambda}_1+\hat{\mu}_1 = \frac{1}{2}\tilde{q}_{MA} = -\frac{1}{2}\tilde{q}_{MM}$$

so that our reciprocity ratios are ratios of these q's:

$$\hat{\rho}_1 = 2\tilde{q}_{AM}/\tilde{q}_{NA}$$

$$\hat{\rho}_2 = 2\tilde{q}_{MA}/\tilde{q}_{AN} \quad .$$

Hallinan examines the following functions of the elements of $\hat{\underset{\sim}{Q}}$, the logarithm of $\hat{\underset{\sim}{P}}$

$$-\frac{1}{\hat{q}_{MM}} = \text{Estimate of stability of Mutual Dyad}$$

- $\frac{1}{\tilde{q}_{AA}}$ = Estimate of stability of Asymmetric Dyad

- $\frac{1}{\tilde{q}_{NN}}$ = Estimate of stability of Null Dyad

- $\frac{\hat{q}_{AM}}{\tilde{q}_{AA}}$ = P{transition A$\Rightarrow$M|transition occurs}

- $\frac{\hat{q}_{AN}}{\tilde{q}_{AA}}$ = P{transition A$\Rightarrow$N|transition occurs}

Assuming that the $\hat{q}$'s are identical to the $\tilde{q}$'s[8], we can easily see that the stability rates are simply reciprocals of our parameters:

- $\frac{1}{\tilde{q}_{MM}} = 2(\hat{\lambda}_1 + \hat{\mu}_1)^{-1}$

- $\frac{1}{\tilde{q}_{AA}} = (\hat{\lambda}_0 + \hat{\lambda}_1 + \hat{\mu}_0)^{-1}$

- $\frac{1}{\tilde{\hat{q}}_{NN}} = 2\hat{\lambda}_0^{-1}$

Consequently, there is no new information contained in the stability rates. In addition, if we write the conditional probabilities of asymmetric change as $\hat{q}_{AM}/(\hat{q}_{AM} + \hat{q}_{AN})$ and $\hat{q}_{AN}/(\hat{q}_{AN} + \hat{q}_{AM})$, we see that the numerators, equivalent to $\hat{\lambda}_0 + \hat{\mu}_0$ and $\hat{\lambda}_1$ respectively, are being "compared" to the denominator $\hat{\lambda}_0 + \hat{\lambda}_1 + \hat{\mu}_0$. While this comparison adds something to our knowledge of dyadic change, we believe that more instructive are $\hat{\rho}_1$ and $\hat{\rho}_2$ which are ratios of $2\tilde{q}_{AM}$ to $\tilde{q}_{NA}$, and $2\tilde{q}_{MA}$ to $\tilde{q}_{AN}$. In short, the traditional interpretation of Markov infinitesimal transitional probabilities is appropriate in the absence of a structural model; but when we have additional information about the elements of $\underset{\sim}{Q}$, in the form of structural parameters, it should be used to draw substantive conclusions.

It is useful to additionally confirm the model by deleting a pair of consecutive observations from the calculation of the pooled $\hat{\underset{\sim}{P}}$. Call this matrix $\hat{\underset{\sim}{P}}^*$. One then estimates $\underset{\sim}{Q}$ from this $\hat{\underset{\sim}{P}}^*$, and compares the empirical probability transition matrix computed from just the pair of deleted observations, $\hat{\underset{\sim}{P}}(t_{i+1} - t_i)$, with the predicted $e^{(t_{i+1}-t_i)\tilde{\underset{\sim}{Q}}^*}$, where $\tilde{\underset{\sim}{Q}}^*$is the MLE of $\underset{\sim}{Q}$ computed from $\hat{\underset{\sim}{P}}^*$. This should be done for all consecutive observations, yielding (n-1) confirmatory checks. This is a form of cross-validation that is quite popular in regression analyses.

Conclusions

In these last paragraphs, we will discuss two crucial issues that are very much unresolved: 1) the sensitivity of longitudinal analyses to the magnitude of measurement error present in the data and to the set of out-degrees, the number of choices made by each individual; and 2) the effect of population heterogeneity on the model. Many of the problems mentioned in the previous pages can be solved with a variety of statistical methods and hypothesis tests. The two that we now grapple with are much more difficult and still open to investigation. Most of what we say in this section is in the form of questions that we pose in the hope that some band of adventurous researchers will tackle them.

How many of the observed changes that occur in a friendship network over time are real changes, and not just artifacts of the data collection process? What effect do these false changes have on the longitudinal analysis? More research is needed on the resistance of the common sociometric techniques to noise and measurement error. Holland and Leinhardt's (1973) well-cited study of measurement error is all we know at present about how one can be led astray in reaching substantive conclusions, particularly if data are fixed-choice.

How sensitive are our estimates of $\underset{\sim}{Q}$ to the magnitude of the outdegrees and the density of the network? In a network with few choices, there are many null dyads. The finding that asymmetric dyads are much more likely to change into null dyads than mutual dyads may be due not to any sociological property of the group, but simply to its low choice density. It may be very difficult to reciprocate a choice in such a network.

The second concern that we have is the effect of differential popularity on the dyadic transition rates. This population heterogeneity may cause asymmetric choices to predominate, and not change into mutuals or nulls over time. Katz and Proctor (1959) found heterogeneity in their longitudinal study in the form of a sex cleavage among the students in their sixth-grade classroom. Our popularity model (Wasserman, 1979a) allows one to study the effect of popularity on individual choices, independently of reciprocity effects. Unfortunately, a blend of the popularity and reciprocity models is mathematically intractible.

Another way to study popularity and reciprocity simultaneously is through the use of exogenous variables for nodal characteristics. Tuma, Hannan, and Groeneveld (1979) regress the transition rates of continuous-time Markov process on a set of exogenous carrier variables. Fienberg and Wasserman (1979) are developing a method for the introduction of such variables into a categorical data set of dyadic or triadic interactions on one or more sociometric generations. Both are promising, although indirect, methods of studying the effects of population heterogeneity.

Footnotes

1) Both Hallinan (1978) and Wasserman (1978a) review other longitudinal analyses of social networks.

2) Sorensøn and Hallinan (1976) and Hallinan (1978) succumb to some of these pitfalls. We will not point out specific instances in this note.

3) The mean waiting time in state i is $-1/q_{ii}$, and the probability of a transition to state j, given that a transition occurs, is $-q_{ij}/q_{ii}$, where the q's are elements of $\underset{\sim}{Q}$, the intensity matrix of the process. See Karlin and Taylor (1975) for additional information.

4) Fienberg and Wasserman (1979) are currently developing log-linear models to analyze such data. The key is to treat the matrices not as Markov transition matrices, but simply as contingency tables with a special structure.

5) We use the term "exploratory" in much the same way as Tukey (1977), although we have an implicit, stochastic mechanism guiding our exploration, and thus are not totally free from parametric assumptions.

6) $\hat{\underset{\sim}{P}}(s,t)$ is the maximum likelihood estimate of $\underset{\sim}{P}(s,t) = e^{(t-s)\underset{\sim}{Q}}$ for stationary continuous-time Markov chains, $s<t$. It is computed simply by standardizing the empirical transition matrices (Table 1) by dividing by the row totals.

7) Estimation and testing longitudinal data in the form of contingency tables is the subject of Chapter 7 of Bishop, Fienberg, and Holland (1975).

8) Of course, the $\tilde{q}$'s are more appropriate (and consequently estimate the correct quantities) than the $\hat{q}$'s since the former are maximum likelihood estimates under the model, which constrains q_{NM} and q_{MN} to be zero.

References

Anderson, T.W. and Leo A. Goodman (1957) "Statistical inference about Markov chains." Annals of Mathematical Statistics, 28:89-110.

Billingsley, Patrick (1961) "Statistical methods in Markov chains." Annals of Mathematical Statistics, 32:12-40.

Bishop, Yvonne M.M., et. al. (1975) Discrete Multivariate Analysis: Theory and Practice. Cambridge, MA: The MIT Press.

Fienberg, Stephen and Stanley Wasserman (1979) "Categorical data analysis of directed graphs, with applications to sociology." Presented at 1979 American Statistical Association Annual Meeting.

Galaskiewicz, Joseph and Stanley Wasserman (1979) "A study of change in a regional corporate network." Technical Report #347, School of Statistics, University of Minnesota. Presented at 1979 American Sociological Association Annual Meeting.

Hallinan, Maureen T. (1978) "The process of friendship formation." Social Networks, 1:193-210.

Holland, Paul W. (1977) "Analysing sociometric data." International Encyclopedia of Neurology, Psychiatry, Psychoanalysis and Psychology, edited by B. Wolman.

Holland, Paul W. and Samuel Leinhardt (1973) "The structural implications of measurement error in sociometry." Journal of Mathematical Sociology, 3:85-111.

Holland, Paul W. and Samuel Leinhardt (1977) "A dynamic model for social networks." Journal of Mathematical Sociology, 5:5-20.

Karlin, Samuel and Howard Taylor (1975) A First Course in Stochastic Processes, Second Edition. New York: Academic Press.

Katz, Leo and Charles H. Proctor (1959) "The concept of configuration of interpersonal relations in a group as a time-dependent stochastic process." Psychometrika, 24:317-327.

Keiding, Neils (1974) "Estimation in the birth process." Biometrika, 61:71-80.

Keiding, Neils (1975) "Maximum likelihood estimation in the birth-and-death process." The Annals of Statistics, 3:363-372.

Miller, Rupert G. (1977) "Developments in multiple comparisons, 1976-1977." Journal of the American Statistical Association, 77:779-788.

Newcomb, Theodore M. (1961) The Aquaintance Process. New York: Holt, Rinehart & Winston.

Singer, Burton and Joel E. Cohen (1978) "Event rates from panel surveys and continuous-time Markovian models, with applications to malaria." Technical Report #15, Center for the Social Sciences, Columbia University.

Singer, Burton and Seymour Spilerman (1974) "Social mobility models for heterogeneous populations." Sociological Methodology, 1973-1974, edited by Herbert Costner. San Francisco: Jossey-Bass, 356-401.

Singer, Burton and Seymour Spilerman (1976) "The representation of social processes by Markov models." American Journal of Sociology, 82:1-54.

Singer, Burton and Seymour Spilerman (1977) "Trace inequalities for mixtures of Markov chains." Advances in Applied Probability, 9:747-764.

Singer, Burton and Seymour Spilerman (1978) "Clustering on the main diagonal in mobility matrices." Sociological Methodology 1979, edited by Karl Schuessler. San Francisco: Jossey-Bass, 172-208.

Sorensøn, Aage B. and Maureen T. Hallinan (1976) "A stochastic model for change in group structure." Social Science Research, 5:43-61.

Tukey, John W. (1977) Exploratory Data Analysis. Reading, Mass: Addison-Wesley.

Tuma, Nancy B. et.al. (1979) "Dynamic analysis of event histories." American Journal of Sociology, 84:820-854.

Wasserman, Stanley (1978a) "Models for binary directed graphs and their applications." Advances in Applied Probability, 10:803-818.

Wasserman, Stanley (1978b) "Distinguishing between stochastic models of heterogeneity and contagion." Technical Report #332, School of Statistics, University of Minnesota.

Wasserman, Stanley (1979a) "Analysing networks as stochastic processes." submitted to Journal of the American Statistical Association.

Wasserman, Stanley (1979b) "A stochastic model for directed graphs with transition rates determined by reciprocity." Sociological Methodology 1980, edited by Karl Schuessler. San Francisco: Josey-Bass, in press.